To Professor A. Almeida Costa
With compliment of author
Kiyoshi Iseki
June 27. 1950.

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On definitions of topological space.

By KIYOSI ISEKI

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The object of this note is to give the conditions to be equivalent to the Kuratowski's topological space with the closure. [1]

By a topological space R is meant a space, in which a closure operation is defined i. e., to each subset X of R, set $\overline{X}$ of R, the closure of X, is associated satisfying the condition:

(1) $\quad X \subset \overline{X}$

(2) $\quad \overline{\overline{X}} = \overline{X}$

(3) $\quad \overline{X \cup Y} = \overline{X} \cup \overline{Y}$

It is easily seen that (3) implies isotone. [2] Since $X \subset Y$ means $X \cup Y = Y$, using (1) it implies $\overline{X} \cup \overline{Y} = \overline{Y}$, whence $\overline{X} \subset \overline{Y}$. this shows that $X \subset Y$ implies $\overline{X} \subset \overline{Y}$

THEOREM 1. *The conditions* (1)-(3) *on topological space R are equivalent to the only one condition*; [3]

(4) $\quad Y \cup \overline{Y} \cup \overline{\overline{X}} = \overline{X \cup Y}$

PROOF. The condition (4) holds if R is topological space above:

$$Y \cup \overline{Y} \cup \overline{\overline{X}} = \overline{Y} \cup \overline{\overline{X}} = \overline{X} \cup \overline{Y} = \overline{X \cup Y}$$

Conversely, if R is a system with the closure satisfying (4), then we have

$$Y \cup \overline{Y} \cup \overline{\overline{Y}} = \overline{Y}$$

This means $Y \subset \overline{Y}$, $\overline{\overline{Y}} \subset \overline{Y}$, therefore $Y \subset \overline{Y}$, $\overline{\overline{Y}} = \overline{Y}$.

Using these results, (4) implies

$$\overline{X \cup Y} = Y \cup \overline{Y} \cup \overline{\overline{X}} = \overline{Y} \cup \overline{X}$$

We note here, only the following result.

THEOREM 2. *Every Boolean algebra with conditious* (1)-(3) *under the closure operation* $X \longrightarrow \overline{X}$ *are equivalent to*

$$Y \cup \overline{X} \cup \overline{\overline{X}} = \overline{Y \cup Y}.$$

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1) C. Kuratowski; *Topologie* I (1933) p. 15, or Fund. Math. vol. 3 (1922)

2) G. Birkhoff; *Lattice theory* (1948) p. 3.

3) G. Birkhoff, loc. cit. p. 50, or J. Ridder, *Einige Anwendungen des Dualitätsprinzips in topologischen Strukturen,* Verhand. Ned. Akad., Amsterdam vol. 50 p. 341 (1947) I can not see the paper by Monteiro quoted in them.

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To Professor A. A. Costa
Thank you for your reprint on quasi-regular ideal
20/3-51.

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A construction of two-valued measure on Boolean algebra.

By KIYOSHI ISEKI

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The object of this note is to give a direct construction of two-valued measure on infinite Boolean algebra[1]. Stone's theorem on the existence of prime ideal (or ultrafilter) is equivalent to this result[2]. Therefore the existence can be proved easily with the help of Hausdorff maximality principle.[3] Quite recently R. Sikorski[4] has given a interesting proof.

A two valued measure on Boolean algebra L is a function $m(x)$ defined on every element x of the L, satisfying the following conditions:

1. $m(x)$ takes only two values 0 and 1.
2. For two disjoint elements x, y, $m(x \smile y) = m(x) + m(y)$. (finitely additive)
3. For unit 1 of L, $m(x) = 1$.

We get the following result.

THEOREM. *There exists at least one two-valued measure on any infinite Boolean algebra. Moreover, for a given non-zero element x (or $x \neq 1$), there is two valued measure which satifies $m(x) = 1$ (or $m(x) = 0$).*

The idea of the proof goes back to W. Siespinski[5].

A subset A of any lattice L (not neeessarily Boolean algebra) is said to have

1) For definition of Boolean algebra, see G. Birkhoff, *Lattice theory* Revised Edition (1948) Ch. X. We follow here the notations of his *Lattice theory*

2) Cf. E. Marczewski, *Two-valued measure and prime ideal in field of sets* C. R. de Varsovie III (1947) PP. 11-17.

3) For detail, *The Hausdorff maximality principlo*, printed by The Tulane University of Louisiana.

4) R. Sikorski, *A theorem on extension of homomorphisms*, Annales Soc. Pol. Math. 21 (1948) pp. 332-335.

5) W. Sierpinski, *Un theoreme sur les familles d'ensembles et ses applications*, Fund. Math. 33 (1945) pp. 1-6.

Herrn Prof. Dr. A. Almeida Costa
mit herzlichen Grüßen
A. Kertész

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